LOCAL VALUE

IN

CHINESE ARITHMETICAL NOTATION

BY

J. EDKINS, D. D.

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The principle of local value in arithmetical notation was introduced to Europe at the beginning of the thirteenth century, by Leonard of Pisa in a work called Liber Abbaci, Book of the Abacus. Such is the statement of Cyclopaedias. Before this time the Persians and Arabs had possessed a knowledge of this mode of writing numbers, and it was by them attributed to the Hindoos. In Arabic or Persian Bibles the verses are marked on the right hand margin. Ten is expressed by a down stroke and a dot on the right hand. Twenty one is marked by two and a down stroke on the right for one. If a three is written with a five on the right it is to be read 35. Such is the principle of local value as practised by the Arabs and Persians. A separate symbol is used for two, for three, for four, and so on up to nine, and there the Arabs and we adopt the principle of local value. To write ten we use one on the left and a cypher or a dot on the right. We do the same for all the multiples of ten however far we may proceed. Europe has used this very convenient notation for not quite seven hundred years.

In Chinese commerce the same principle is in use in writing, but there is this difference. Strokes are used instead of special symbols for 1. 2, 3, 6, 7 and 8. Thus two strokes are two, and three strokes are three. When four is reached the Chinese merchant writes 〤 and for five 〥. Beyond that be returns to strokes. He writes a short down stroke and places it at right angles to a cross stroke. This is six. He adds a cross stroke to six to make seven 〧, two to make eight 〨, and a new symbol to make nine 〩. This mode of numeration is known as Soochow Matsï. The city of Soochow has been for centuries the chief place of trade in China and it is trade that has maintained during a long period this mode of writing numbers. The writer of the matsï, begins with a down stroke, and changes to crossstrokes when he comes to six.

The connection of the matsï vith the common Chinese abacus is very close. The abacus is a picture of the mode of working the matsï. If the calculator brings up three balls to the cross bar and brings down one ball upon it in the upper division of the abacus, he does just what he would do if he made an upright stroke with his pencil at right angles to three horizontal strokes, and it is read eight in each case. The matsï writing some one might say is a written representation of the process with the abacus. It does not much matter which way this is put. The abacus is simply a new instrument intended to help in calculation, and does not contain any new princi ple. The stringing of balls on wires is certainly an idea brought from the west, for the Greeks and Romans had it. The advantage of this device is that the mind is saved by it from the need of recollecting complicated numbers. Then as to the principle of local value according to which a ball on each wire counts as ten times a ball on the wire to the right of it, it was certainly not new. It came from the calculating slips called ch'eu 籌, which have been in use from ancient times in China. These were made of wood or bamboo and they are very frequently mentioned in the histories, and other works from the Yi li and the Shï-Ki downwards.

These counters 籌 were rather more than, four inches long, and lay before the calculator upon a table or counter. They were placed lengthways for units as far as five and then horizontally. One upri ht slip coming down at right angles on a horizontal or cross slip was six. Another cross slip was added for seven, a second for eight, a third for nine. But the calculator had the option of placing his counter horizontally with another perpendicular to it below touching its centre, for six. A cross bar with two perpendicular counters below was seven. Eight had three upright counters. When they came to ten the calculators put a single counter upright on the left, using the principle of local value. A counter on the left is always ten times what it is on the right. This being the mode of working the counters, when the process was written down, an upright stroke descending upon a cross stroke was thus a symbol for five, or it might be written across, and six was either 𝍮 or 𝍥· Eight was 𝍰 or 𝍧. Authors on mathematics liked to change the mode, and we find 7,632 written 𝍯 𝍥 𝍫 𝍡. But they always as a rule began with a down stroke. Mathematical learning was very flourishing in the Sung dynasty and many able calculators have left behind them works where numbers written with this notation are mixed with the common Chinese numerals. These writers also use 〤 for four occasionally and 〩 for nine, but for 6. 7. 8. they prefer strokes either perpendicular or horizontal at the discretion of the writer.

The counters called Ch'eu seem to have been in use for about fourteen hundred years. The well known native mathematician Mei wen ting has in his little work 古算器考 Ku swan c'hi k'au « Inquiry into the implements used for calculating in ancient times » described the Ch'eu among others. This was not the only name by which wooden slips used for arithmetical calculation were known. The verb 算 Swan calculate was also employed as a noun to denote bamboo slips used as counters. The old form of writing swan was 祘 This character consists of two twos and two threes and suggests its own meaning, being without much doubt a picture of the old counters in position. Mei wen ting inquires whether these old slips used as counters were borrowed from the divining sticks of the Yiking. He hesitates to adopt this idea because the sticks of the diviners were from 2 to 5 feet in length, were only 49 in number and too long to be used horizontally. They do not appear to have been employed except in times of doubt and they were not in common use. The arithmeticians of all ages relied on counters and they were arranged from left to right on the decimal sytem, as in the abacus and as in the European mode of writing numbers.

Mei wen ting then proceeds to cite the Tso chwen in which, under the year, B. C. 542 it is stated that the divining officer Chau on one occasion said the character 亥 hai is composed of two at the head and of sixes in the body. If you take the symbol jor two and place it beside the sixes you obtain the number of days that this man has lived. This was said in reference to an old man of seventy three who came and took his place at a state feast in Shansi. Some who were present asked him to tell his age. He replied, « the « year in which I was born began with a new moon upon a day of « which the cyclic characters were 甲子 Kia tzï. Since my birth « 445 Kia tsï days have passed and the last was 20 days ago. » The interrogators went to the palace to inquire when he was born and it was then that the diviner of the Tsin country gave the answer respecting the chaaracter Hai. He had lived 26660 days.

From this incident we learn that men in those times took account of their ages in cycles, including, both the cycle of twelve, and that of ten. They also for six used a short down stroke coming at right angles upon a cross stroke or a cross stroke above a down stroke. They also made use of the principle of local value and this was from left to right, for the Chinese always write each character from left to right beginning at the left hand upper corner.

It is certainly by a very remarkable destiny that this instance of arithmetical writing from left to right has been preserved. Without it no one could have ventured to defend the opinion that the Chinese so long ago wrote numbers from left to right according to the principle of local value. If any one had been bold enough to say this many critics would have charged him with rashness. But here we have the proof clearly given.

I may state here in passing that before the abacus came into favour in the 14 th century, the counting slips called c'heu were in use for a period not less than fourteen centuries and probably more. That kind of bamboo slip called c'hien 籖 used for casting lots in Tauist temples was also employed in part during the same period to count with.

The most probable explanation of this early use of local value in arithmetical notation is that it was of foreign origin.

In Saycè's Astronom yand Astrology of the Babylonians published separately from the Journal of the society of Biblical Archaeology there are many instances of writing in cuneiform where the same principle is adopted. Thus 𒌋𒐈 is 14, 𒌋𒐉 is 15, 𒌋𒐊 is 16, 𒎙𒁹 is 21, 𒎙𒈫 is 22, 𒐈 is 3, 𒎙𒐋 is 27, 𒎙𒐌 is 28, 𒈫 is 2, 𒐈 is 4.

The same author in his lectures on the Assyrian Language, says the Assyrians generally made use of a symbolical mode of expressing numbers like our cyphers. In this system an upright wedge denoted one, two upright wedges two and so on. For four instead of writing the wedges one after the other three were written in one line and one beneath thus 𒐈 ; and the same arrangement was adopted as far as seven, when a third line was added 𒁹𒁹𒁹𒁹𒁹𒁹𒁹. For nine there was besides 𒁹𒁹𒁹𒁹𒁹𒁹𒁹𒁹𒁹 the abbreviation ⸜ Ten was expressed by 𒌋 and the succeeding numerals were denoted by the help of this arrow head and the wedge. Thus eleven was 𒌋𒁹, 15 𒌋𒁹𒁹𒁹𒁹𒁹, 20 𒌋𒌋 and so on.

Professor Sayce proceeds to speak of the symbol for 60. The Accadians had attained great mathematical proficiency and had found that the duodecimal was a more convenient numerical system than the decimal. They consequently made 60 their unit, and represented both one and sixty by the same upright wedge. It was made thicker and larger usually, when it signified sixty, but it is sometimes not easily distinguished from one. After 60 there is no difficulty 𒁹𒁹 sixty one, and 𒁹𒁹𒁹 sixty two are plain enough; so also 𒁹𒌋𒌋 eighty.

In this Babylonian system as thus described we have the principle of local value as in our Arabic numerals. A number on the left in any column is ten times what it is on the right. Then also the cardinal numbers are expressed as by the Chinese counting slips. An upright stroke marks one, two upright strokes are two, three upright strokes are three. At this point the Babylonian and Chinese systems diverge. The signs for four, five etc. up to nine consist of as many strokes placed in two or three rows. This is the Babylonian method. In the Chinese method as already described upright strokes are used as far as to five. Six is formed by a small down stroke upon a cross stroke meeting it in the centre. A cross stroke is added for seven, two for eight, three for nine. The divergence may have been not real. We have only the numerals of Sargon's library. If we knew the commercial writing used by Babylonian merchants it might bo found to resemble the Chinese more closely.

The newer Chinese method, that of commerce, represents four by an oblique cross ✕ and this is a running band contraction of ancient origin. This same mark for four is found in the inscriptions on rocks in India of the time of Asoka This circumstance mentioned by Mr Edward Thomas ought not to be lost sight of. It may be a link of perfectly historical validity connecting east and west in regard to the use of signs of numbers which are easily propagated by commercial intercourse from one country to another.

The mark for five in the modern commercial notation of the Chinese is merely a running hand contraction. It is an instance of a change introduced by the Chinese themselves for the purpose of saving time in writing. The mark for nine however is probably a compound of five — and four ✕.

The reasons for assigning the origin of the principle of local value to the Babylonians are the following.

1. The first Chinese example being in the year BC 542 cannot compare in antiquity with the Babylonian usage. In the sixteenth century before Christ the Babylonians had made great attainments in mathematics and bricks of that age show that to find the square and cube root of numbers were problems which they could then master. They have left tables of squares and cubes.

2. There are no traces of Hindoo proficisncy in mathematics as early as the sixth century before Christ. The earliest Hindoo astronomer Aryabatta lived in the fifth century after Christ. India doubtless owed much to Babylonia and the residence of merchants from foreign countries in cities of the coast brought to the Hindoos many successive helps to mental progress, which they silently incorporated, after the Devanagari was invented, into their system of education. The art of writing was at its commencement in India at a time when the Chinese were gathering what help they could from foreign countries in the age before Confucius. At the period of Alexander's conquest and before it, Professor Max Müller tells us, the Indians had becaume acquainted with the art of writing but abstained

from using it for literary purposes. The conclusion drawn from the wide range of facts known to this popular and learned writer being such as this, we may suppose that writing came into India by the sea ports in much the same way as the Buddhist cosmography with its representation of a circular ocean surronnding the world and the Sumeru moumtain in the distant north. Both India and China in those times appear to have received information brought to them by navigators visiting in the way of commerce the various harbours of the Indian seas. The way is thus open for a third argument.

3. Since the Devanagari writing came into Iudia from Western Asia through Phœnician trade, without any recorded statement by Hindoo writers of the fact, it is very probable that the principle of local value in arithmetical notation which was used in China in the sixth century before Christ came into that country also through the visits of traders who then made voyages in the Indian Ocean. A very large portion of the cuneiform tablets unearthed from Mesopotaminn mounds consist of commercial documents because the trade on the Euphrates was very large. The ocean played a great part in the mythology and cosmography of Babylon and Egypt as well as the neighbouring countries and this was because of the extensive commerce in which Babylonian, Phœnician, and Arabian ships were in ancient times engaged. The ability to write in cuneiform or Phoenician or some alphabet based upon them was essential in maritime trade. This is the best way of accounting for the knowledge of the principle of local value by the Chinese in the time of Confucius. They learned it from the same source from which they received the knowledge of the clepsydra, the dial, astronomy and astrology.

4. Auxiliary evidence is at band in the knowledge of the cycle of sixty which had been used to denote days with two characters

since the beginning of the Cheu dynasty, and with one character since the time of the Yellow Emperor, Men counted their age in cycles of days and based astrological calculations as to what would happen upon certain days by observing the place they held in this cycle. It was so in the case of this old man of 73. But we know that the Babylonians also made 60 an important unit in calculation it being divisible both by twelve and by ten, and appearing constantly in chronology and in the computations of astrologers.

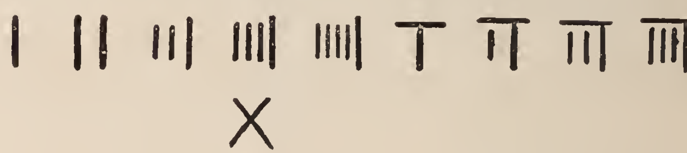
5. The Chinese counting slips were by preference placed upright. When placed across it was a variation. But it is also true of the Babylonian wedges and arrow heads that they are on the whole upright.

These five arguments suffice as proof presumptive that the Chinese numeration of this kind came from the Babylonians.

Cuneiform Numerals B. C. 1600

NUMERALS

of SUNG dynasty used by mathematical authors



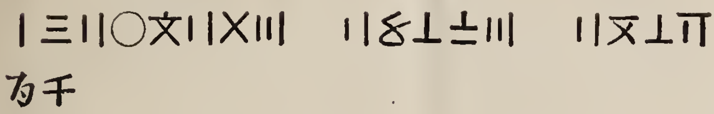
NUMERALS

in other forms from the HAN dynasty downwards

SOOCHOW MATSZ or commercial numerals

Examples of Matsz

13209243 25673 2967



NUMERALS in TSO CHWEN B. C. 542

2 6 6 6 0

HAI 亥 

OLD forms of the character HAI

OLD form of the character 算 *Swan*